Solving the schrodinger eq.

$$\begin{bmatrix} E_0 & -A \\ -A & E_0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = E_0 + A \begin{bmatrix} c \\ d \end{bmatrix}$$

eigenvectors $E_0 a - Ab = E_0 c + Ad$

$$O_2^- \Rightarrow \hat{H} = \begin{pmatrix} E_0 & -A \\ -A & E_0 \end{pmatrix} \Rightarrow \text{eigenvalues } E_1 = E_0 + A, \quad E_2 = E_0 - A$$

$$|E_1\rangle = \frac{1}{\sqrt{2}}\left(|+a\rangle - |-a\rangle\right) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$|E_2\rangle = \frac{1}{\sqrt{2}}\left(|+a\rangle + |-a\rangle\right) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

I method

$$|\psi(t)\rangle = C_+(t)|+a\rangle + C_-(t)|-a\rangle = \begin{pmatrix} C_+(t) \\ C_-(t) \end{pmatrix}$$

$$i\hbar \frac{d}{dt}\begin{pmatrix} C_+(t) \\ C_-(t) \end{pmatrix} = \begin{pmatrix} E_0 & -A \\ -A & E_0 \end{pmatrix} \cdot \begin{pmatrix} C_+(t) \\ C_-(t) \end{pmatrix}$$

$$\begin{cases} i\hbar \dot{C}_+(t) = E_0 C_+(t) - A C_-(t) \\ i\hbar \dot{C}_-(t) = -A C_+(t) + E_0 C_-(t) \end{cases}$$

$X = C_+ + C_- \Rightarrow C_+ = \frac{1}{2}(X + Y)$

$Y = C_+ - C_- \Rightarrow C_- = \frac{1}{2}(X - Y)$

$$i\hbar(\dot{C}_+ + \dot{C}_-) = E_0(C_+ + C_-) - A(C_+ + C_-)$$

$$i\hbar \dot{X} = E_0 X - AX \rightarrow X(t) = X(0) \cdot e^{\frac{-i(E_0 - A)t}{\hbar}}$$

if subtract

$$i\hbar \dot{Y} = (E_0 + A)Y \rightarrow Y(t) = Y(0)\, e^{\frac{-i(E_0 + A)t}{\hbar}}$$

$$C_+(t) = \frac{1}{2}(X + Y) = \frac{1}{2} e^{-iE_0 \frac{t}{\hbar}} \left(X(0) e^{iA\frac{t}{\hbar}} + Y(0) e^{-iA\frac{t}{\hbar}} \right)$$

$$C_-(t) = \frac{1}{2} e^{-iE_0\frac{t}{\hbar}} \left(X(0) e^{iAt/\hbar} - Y(0)\, e^{-iAt/\hbar} \right)$$

suppose $|\psi(0)\rangle = |-a\rangle$

$|-a\rangle = C_+(0)|+a\rangle + C_-(0)|-a\rangle$

$C_+(0) = 0, \; C_-(0) = 1$

$X(0) = 1, \; Y(0) = -1$

$$C_+(t) = \frac{1}{2} e^{\frac{-iE_0 t}{\hbar}} \left(e^{iAt/\hbar} - e^{-iAt/\hbar} \right) = i e^{\frac{-iE_0 t}{\hbar}} \sin\left(A\frac{t}{\hbar}\right)$$

$$C_-(t) = e^{-iE_0 t/\hbar} \cos\left(At/\hbar\right)$$

II

$c_1 \frac{1}{\sqrt{2}}|+a\rangle - \frac{c_1}{\sqrt{2}}|-a\rangle + \frac{c_2}{\sqrt{2}}|+a\rangle + \frac{c_2}{\sqrt{2}}|-a\rangle$

$$|\psi(0)\rangle = c_1|E_1\rangle + c_2|E_2\rangle$$

$$\overset{c_1}{\left(\frac{c_1+c_2}{\sqrt{2}}\right)}|+a\rangle + \overset{c_2}{\left(\frac{-c_1+c_2}{\sqrt{2}}\right)}|-a\rangle \checkmark$$

$$|\psi(t)\rangle = c_1 e^{-i(E_0+A)\frac{t}{\hbar}}|E_1\rangle + c_2 e^{-i(E_0-A)\frac{t}{\hbar}}|E_2\rangle \quad |\psi(0)\rangle = |-a\rangle$$

$\langle E_1|-a\rangle = c_1\langle E_1|E_1\rangle + c_2\langle E_1|E_2\rangle$

$$|-a\rangle = c_1|E_1\rangle + c_2|E_2\rangle \Rightarrow c_1 = \langle E_1|-a\rangle = -\frac{1}{\sqrt{2}}$$

$$c_2 = \langle E_2|-a\rangle = \frac{1}{\sqrt{2}}$$

$E_1 = \frac{1}{\sqrt{2}}(|+a\rangle - |-a\rangle)$

$$|\psi(t)\rangle = -\frac{1}{\sqrt{2}} e^{-iE_0 t/\hbar}\left(e^{-iAt/\hbar}|E_1\rangle - e^{iAt/\hbar}|E_2\rangle\right) \checkmark$$